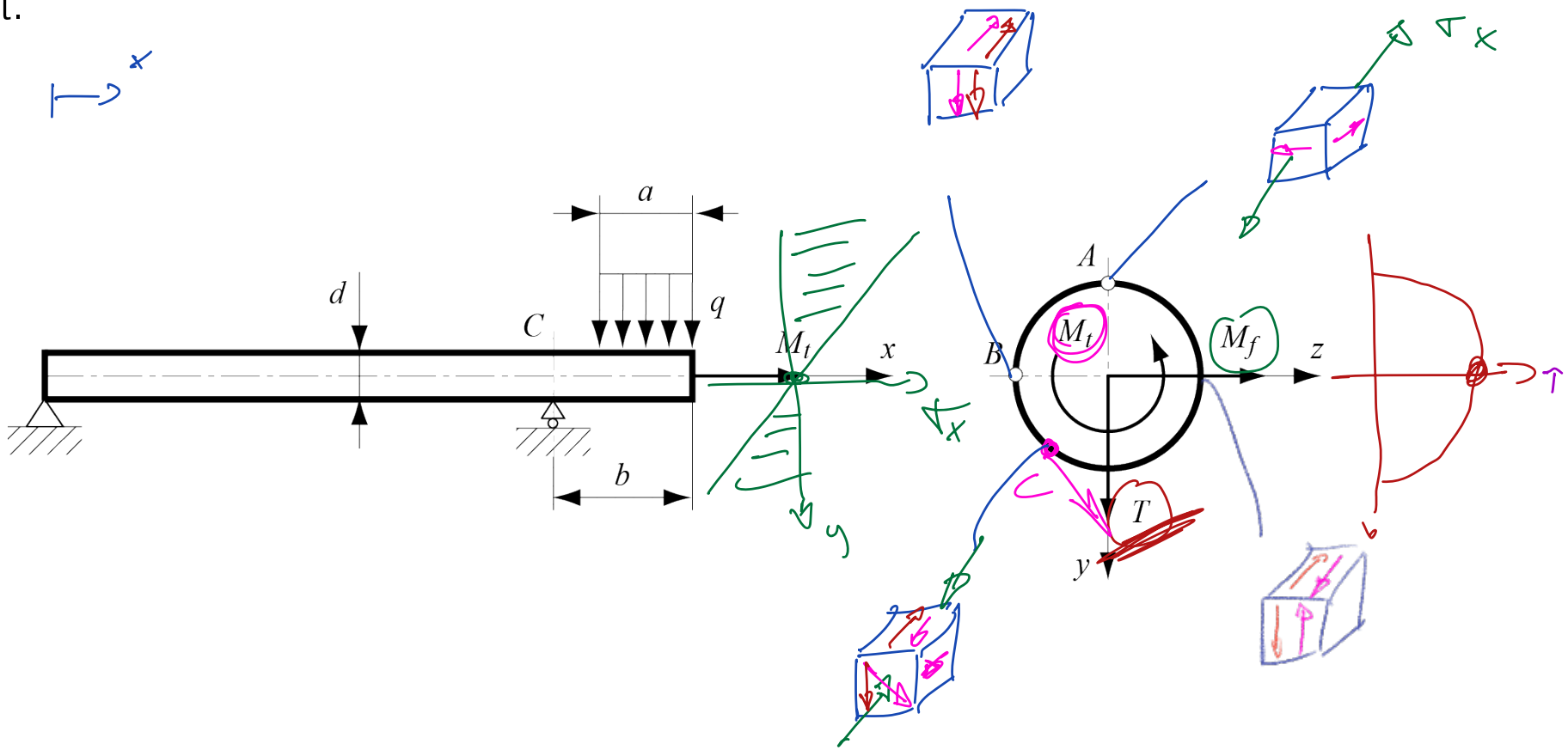
Exemple de problème mécanique

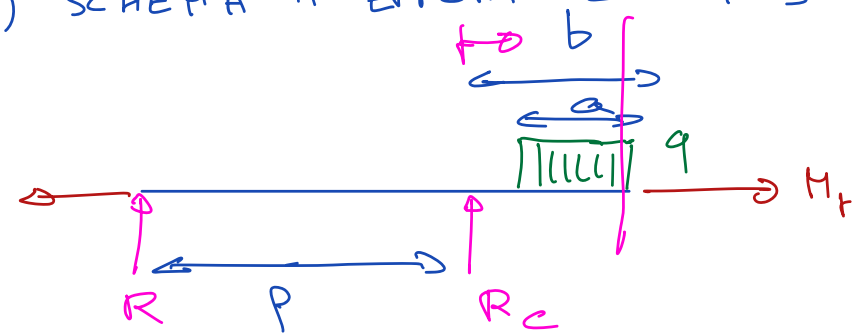
Problème 15.1 et 15.2

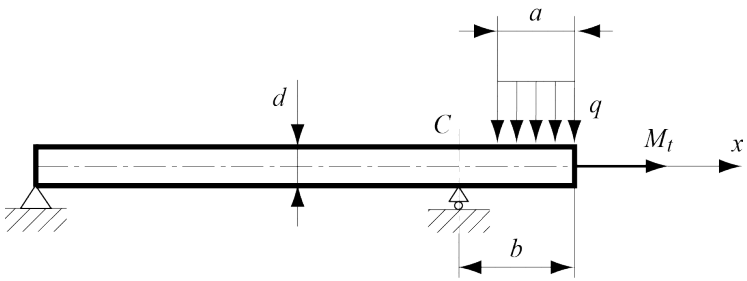
La force de contact d'un engrenage, réduite à l'axe neutre de l'arbre porteur, donne lieu à une force répartie verticale q et un moment de torsion M_t. Identifier la zone pour laquelle l'état de contrainte et le plus critique et calculer les contraintes principales en ce point.



Exemple de problème mécanique

1) SCHEMA + EFFORTS EXTERNES





2) EQUILIBRE FORCES ET MOMENT

$$\sum F_y \qquad R + R_c - qa = 0$$

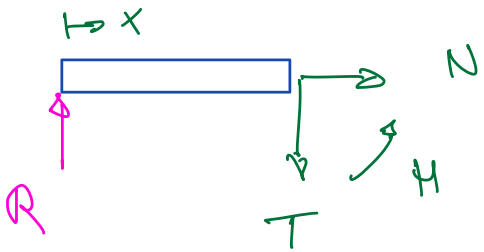
$$\sum M \qquad Rp + \int_{b-a}^{b} qx\,dx = 0$$

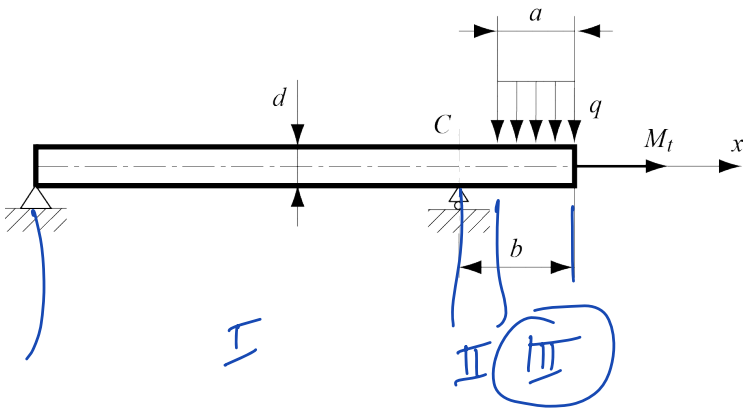
$$\rightarrow \quad R = \frac{-q}{p}\int_{b-a}^{b} x\,dx = \frac{-q}{2p}\left[b^2 - (b-a)^2\right] = \frac{q}{2p}\left(a^2 - 2ab\right)$$

$$\rightarrow \quad R_c = qa - R = \frac{q}{2p}\left(-a^2 + 2ab + 2ap\right)$$

Exemple de problème mécanique

3) DISTRIBUTION EFFORTS INTERIEURS

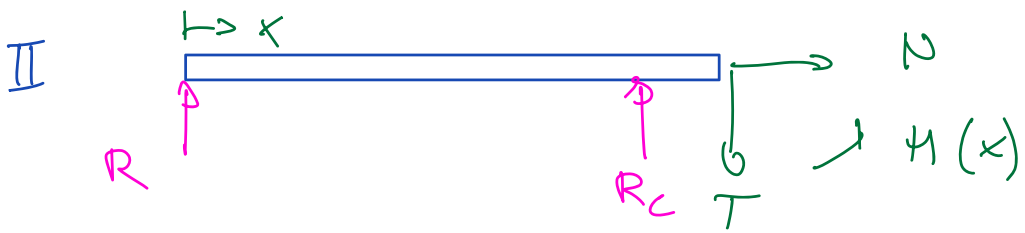




$$T(x) = R$$

$$M(x) = R\,x = \frac{q}{2\ell}\left(a^2 - 2ab\right)x$$

$$M(x=\ell) = \frac{q}{2}\left(a^2 - 2ab\right) \quad \frac{N}{m}\,m^2 = [N\,m]$$



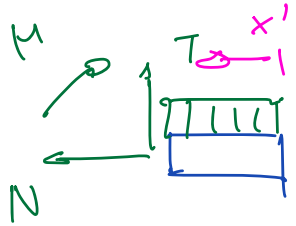
$$T(x) = R + R_C = qa$$

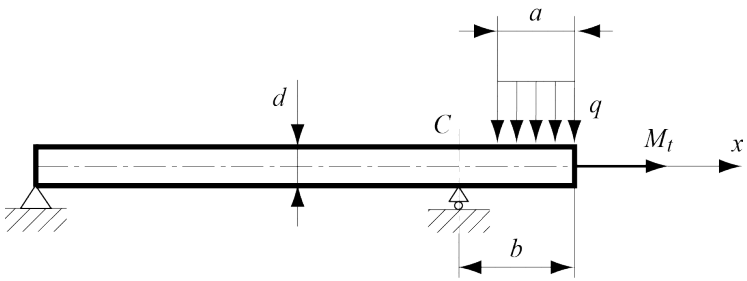
$$M(x) = R\,x + R_C(x-\ell) = qax - \frac{q}{2}\left(-a^2 + 2ab + 2a\ell\right)$$

$$M(x=\ell) = \frac{q}{2}\left(a^2 - 2ab\right)$$

Exemple de problème mécanique

II





$$T(x') = \int_0^{x'} q\,dx = qx'$$

$$T(x'=a) = qa$$

$$M(x') = -\int_0^{x'} qx\,dx = -\frac{1}{2}qx^2$$

$$M(x'=a) = -\frac{1}{2}qa^2$$

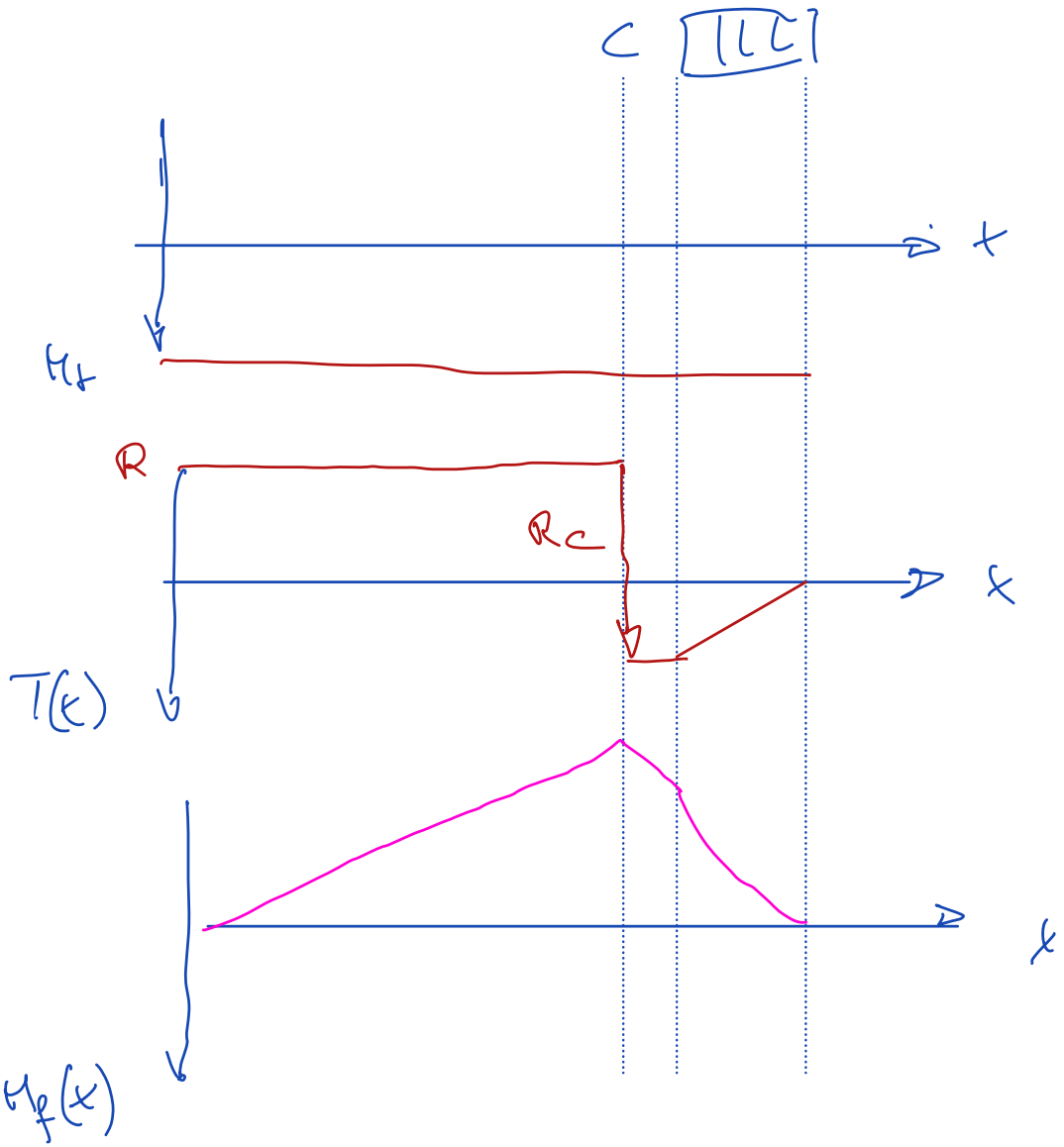
COMPARAISON

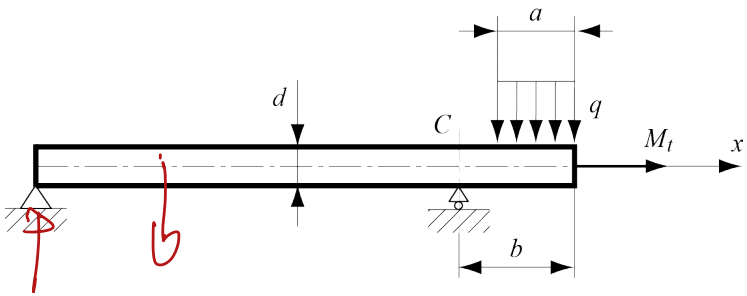
$$M(x = l + b - a) = qax - \frac{q}{2}(-a^2 + 2ab + 2al)$$

$$= \frac{q}{2}(2la + 2ab - 2a^2 + a^2 + 2ab - 2la) = -\frac{qa^2}{2}$$

Exemple de problème mécanique

4) DIAGRAMMES DES EFFORTS





⇒ SECTION CRITIQUE

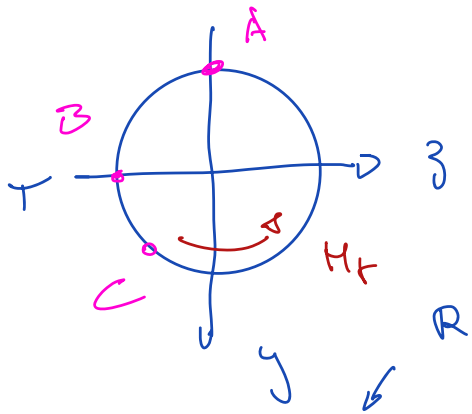
$x = l$

au C

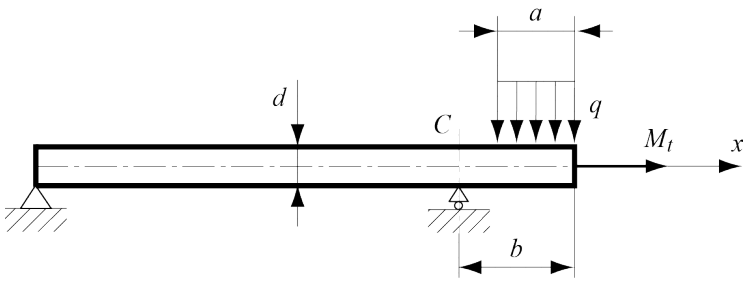
Exemple de problème mécanique

5) TORSION

$I_p = I_y + I_z$



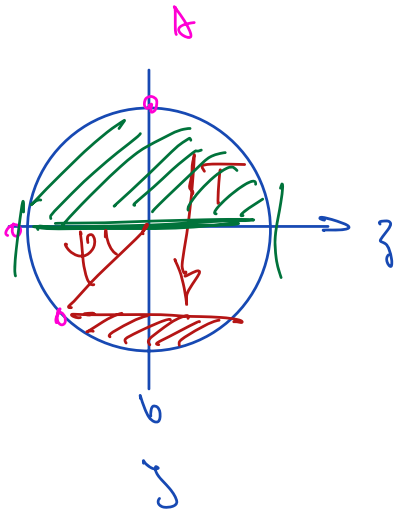
$R = \frac{D}{2}$



$$\tau_{xz}(A) = -\tau_{xy}(B) = -\tau_{x\theta}(C) = -\frac{R M_t}{I_p} = \frac{-16 M_t}{\pi D^3}$$

$I_p = \frac{\pi D^4}{32}$

6) EFFORT TRANCHANT



$$\tau = \frac{T S'}{I_z b}$$

A $b = 0$ $S' = 0$ $\quad \tau_{xy}(A) = 0$

B $b = 2R$ $\quad \tau_{xy}(B) = \frac{4T}{3\pi R^2}$

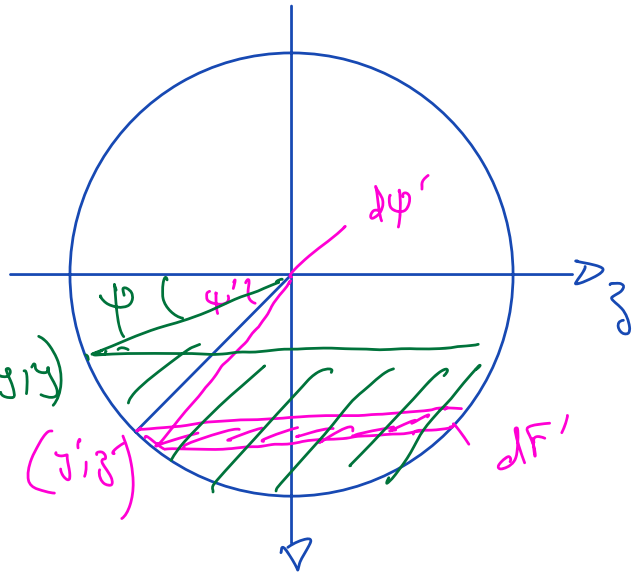
C $b = ?$ $S' =$ $\quad \tau_{xy}(C) = \frac{4T}{3\pi R^2}\cos^2\psi$

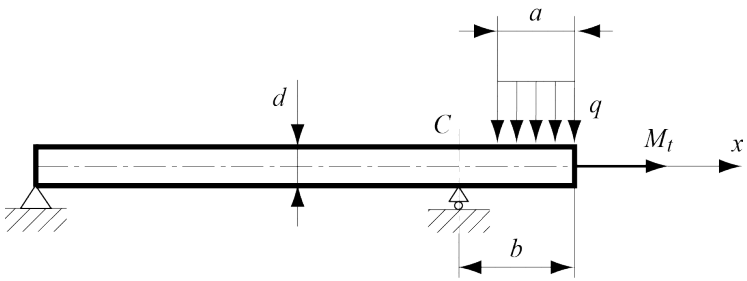
Exemple de problème mécanique

$S' = ?$

$b = ?$

$$\begin{cases} y' = R \sin\psi' \\ z' = R \cos\psi' \end{cases}$$





ELEMENT SURFACE dF'

$$dF' = 2z'\,dy' = 2R\cos\psi' \, R\cos\psi' \, d\psi' = 2R^2\cos^2\psi'\,d\psi'$$

MOMENT STATIQUE

$$S' = \iint_{F'} y'\,dF' = 2R^3 \int_{\psi'=\psi}^{\pi/2} \sin\psi' \cos^2\psi' \, d\psi' = \frac{2}{3} R^3 \cos^3\psi$$

$$b = 2z = 2R\cos\psi$$

$$\tau(\psi) = \frac{TS'}{I_z\, b} = \frac{4T}{\pi R^4}$$

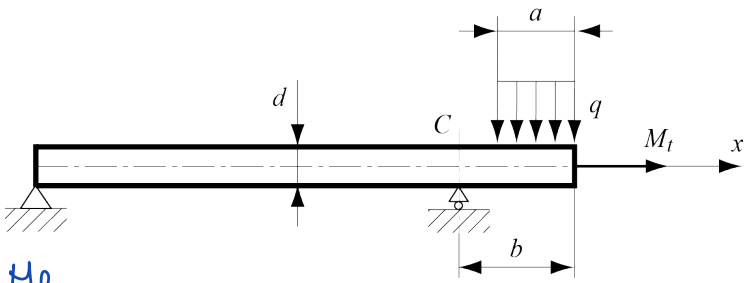
$\frac{\pi R^4}{4}$ ←

$$\frac{\frac{2}{3}R^3\cos^3\psi}{2R\cos\psi} = \frac{4}{3}\frac{T}{\pi R^2}\cos^2\psi$$

Exemple de problème mécanique

7) MOMENT FLEXION

$$M_f = M(x=\rho) = \frac{q}{2\rho}(a^2 - 2ab)$$



$$\tau(y) = \frac{-y M_f}{I_z}$$

A

$$\tau(y=R) = \frac{R M_f}{\frac{\pi R^4}{4}} = \frac{4 M_f}{\pi R^3}$$

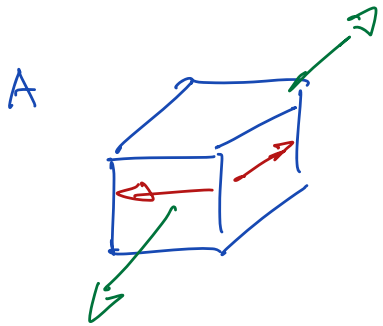
B

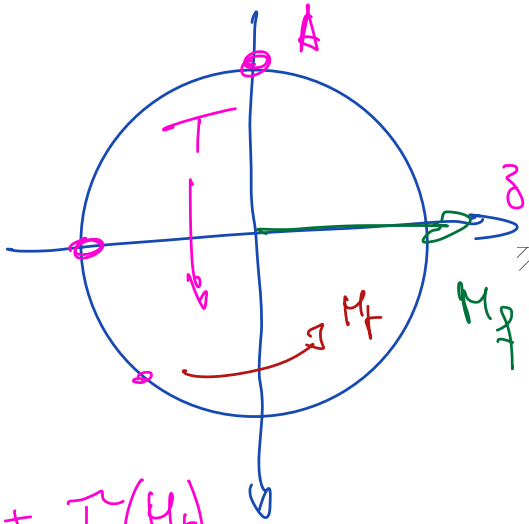
$$\tau(y=0) = 0$$

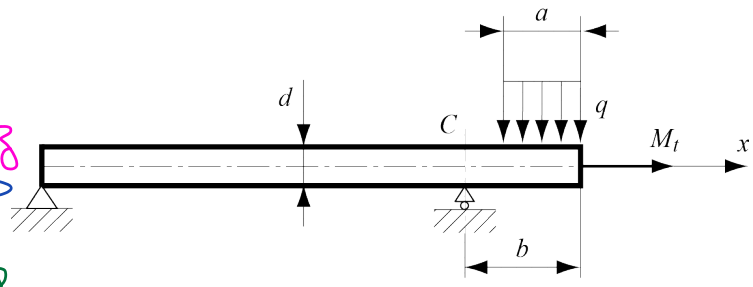
C

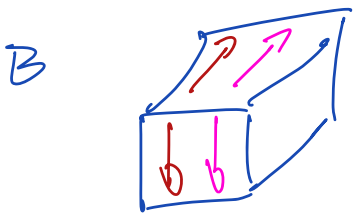
$$\tau(y = R\sin\psi) = \frac{4 M_f}{\pi R^3}\sin\psi$$

Exemple de problème mécanique

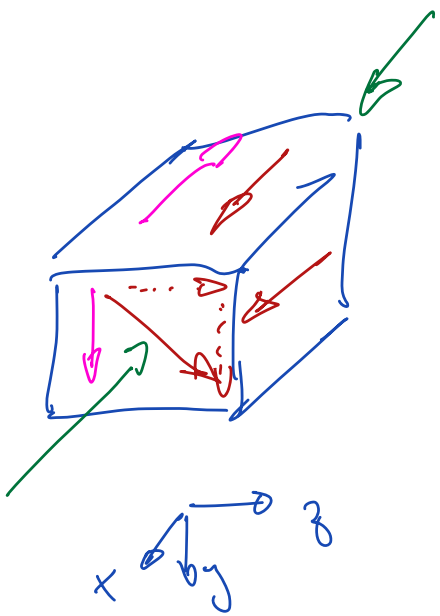








$$\tau = \tau(T) + \tau(M_t)$$



$$\sigma_{1,3}(A) = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2}$$

$$\sigma_{1,3}(B) = \pm \left[\tau(M_t) + \tau(T)\right]$$

$$\sigma_{1,3}(C) = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & 0 & 0 \\ \tau_{xz} & 0 & 0 \end{bmatrix}$$